theory of electricity and magnetism, didn't believe in the existence of fundamental units of charge such as electrons. George Stoney, who at the end of the nineteenth century proposed the electron as a fundamental unit of charge, didn't believe that scientists would ever isolate electrons from the atoms of which they are components. (In fact, all it takes is heat or an electric field.) Dmitri Mendeleev, creator of the periodic table, resisted the notion of valence, which his table encoded. Max Planck, who proposed that the energy carried by light was discontinuous, didn't believe in the reality of the light quanta that were implicit in his own idea. Albert Einstein, who suggested these quanta of light, didn't know that their mechanical properties would permit them to be identified as particles—the photons we now know them to be. Not everyone with correct new ideas has denied their connection to reality, however. Many ideas, whether believed-in or mistrusted, have turned out to be true.

Is there more waiting to be discovered? For the answer to that question, I turn to the all-too-mortal words of George Gamow, the prominent nuclear physicist and science popularizer. In 1945 he wrote, "Instead of a rather large number of 'indivisible atoms' of classical physics, we are now left with only three essentially different entities; nucleons, electrons, and neutrinos . . . Thus it seems that we have actually hit the bottom in our search for the basic elements from which matter is formed." When Gamow wrote this, he had no idea that the nucleons are composites of quarks, which would be discovered within thirty years!

Wouldn't it be strange if we turn out to be the first people for whom the search for further underlying structure ceased to be fruitful? So strange, in fact, that it seems hardly credible? Inconsistencies in existing theories tell us they can't be the final word. Earlier generations had neither the tools nor the motivations of today's physicists for exploring the extra-dimensional arenas that this book will describe. Extra dimensions, or whatever underlies the Standard Model of particle physics, would be a discovery of major importance.

When it comes to the world around us, is there any choice but to explore?

# I

# Entryway Passages: Demystifying Dimensions

You can go your own way.
Go your own way.

Fleetwood Mac

*"Ike, I'm not so sure about this story I'm writing. I'm considering adding more dimensions. What do you think of that idea?"*

*"Athena, your big brother knows very little about fixing stories. But odds are it won't hurt to add new dimensions. Do you plan to add new characters, or flesh out your current ones some more?"*

*"Neither; that's not what I meant. I plan to introduce new dimensions—as in new dimensions of space."*

*"You're kidding, right? You're going to write about alternative realities—like places where people have alternative spiritual experiences or where they go when they die, or when they have near-death experiences?* I didn't think you went in for that sort of thing."*

*"Come on, Ike. You know I don't. I'm talking about different spatial dimensions—not different spiritual planes!"*

*"But how can different spatial dimensions change anything? Why would using paper with different dimensions—11" × 8" instead of 12" × 9", for example—make any difference at all?"*

*"Stop teasing. That's not what I'm talking about either. I'm really planning to introduce new dimensions of space, just like the dimensions we see, but along entirely new directions."*

*Questions I've actually been asked.

L. Randall

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*"Dimensions we don't see? I thought three dimensions is all there are."*

*"Hang on, Ike. We'll soon see about that."*

The word "dimension," like so many words that describe space or motion through it, has many interpretations—and by now I think I've heard them all. Because we see things in spatial pictures we tend to describe many concepts, including time and thought, in spatial terms. This means that many words that apply to space have multiple meanings. And when we employ such words for technical purposes, the alternative uses of the words can make their definitions sound confusing.

The phrase "extra dimensions" is especially baffling because even when we apply those words to space, that space is beyond our sensory experience. Things that are difficult to visualize are generally harder to describe. We're just not physiologically designed to process more than three dimensions of space. Light, gravity, and all our tools for making observations present a world that appears to contain only three dimensions of space.

Because we don't directly perceive extra dimensions—even if they exist—some people fear that trying to grasp them will make their head hurt. At least, that's what a BBC newscaster once said to me during an interview. However, it's not thinking about extra dimensions but trying to picture them that threatens to be unsettling. Trying to draw a higher-dimensional world inevitably leads to complications.

Thinking about extra dimensions is another thing altogether. We are perfectly capable of considering their existence. And when my colleagues and I use the words "dimensions," and "extra dimensions," we have precise ideas in mind. So before taking another step forward or exploring how new ideas fit into our picture of the universe—note the spatial phrases—I will explain the words "dimensions" and "extra dimensions" and what I will mean when I use them later on.

We'll soon see that when there are more than three dimensions, words (and equations) can be worth a thousand pictures.





## *What Are Dimensions?*

Working with spaces that have many dimensions is actually something everyone does every day, although admittedly most of us don't think of it that way. But consider all the dimensions that enter into your calculations when you make an important decision, like buying a house. You might consider the size, the schools nearby, the proximity to places of interest, the architecture, the noise level—and the list goes on. You need to optimize in a multidimensional context, enumerating all your desires and needs.

The number of dimensions is the number of quantities you need to know to completely pin down a point in a space. The multidimensional space might be an abstract one, such as the space of features you are looking for in a house, or it might be concrete, like the real physical space we will soon consider. But when buying a house, you can think of the number of dimensions as the number of quantities you would record in each entry in a database—the number of quantities you find worth investigating.

A more frivolous example applies dimensions to people. When you peg someone as one-dimensional, you actually have something rather specific in mind: you mean that the person has only a single interest. For example, Sam, who does nothing but sit at home watching sports, can be described with just one piece of information. If you felt so inclined, you could picture this information as a dot on a one-dimensional graph: Sam's proclivity to watch sports, for example. In drawing this graph you need to specify your units so that someone else can understand what the distance along this single axis means. Figure 3 shows a plot with Sam as a point along a horizontal axis. This plot represents the number of hours Sam spends per week watching sports on TV. (Fortunately, Sam won't be insulted by this

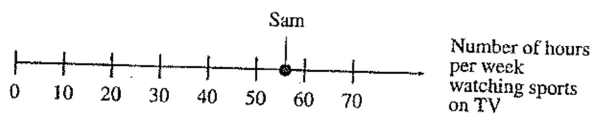


Figure 3. *The one-dimensional Sam plot.*



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example; he is not among the multidimensional readers of this book.)

Let's explore this notion a little further. Icarus Rushmore III (Ike in the above story), a Boston resident, is a more complex character. In fact, he is three dimensional. Ike is twenty-one, drives fast cars, and loses money at Wonderland, a town near Boston with a dog-racing track. In Figure 4 I've plotted Ike. Although I've drawn it on the two-dimensional surface of a piece of paper, the three axes tell us that Ike is definitely three-dimensional.*

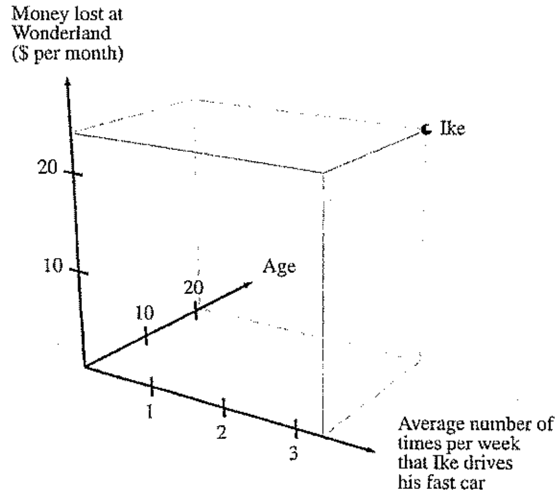


Figure 4. *The three-dimensional Ike plot. The solid notched lines are the coordinate axes of the three-dimensional plot. The point that is labeled Ike corresponds to a 21-year-old boy who loses 24 dollars at Wonderland every month and drives his fast car (on average) 3.3 times a week.*

When we describe most people, however, we usually assign them more than one, or even three, characteristics. Athena, Ike's sister, is

*If you're picky, you'll object that Sam too has an age and therefore another dimension. However, I've assumed that Sam has been the same way for years so his age isn't relevant.





an eleven-year-old who reads avidly, excels at math, keeps abreast of current events, and raises pet owls. You might want to plot this too (though why, exactly, I'm not really sure). In that case, Athena would have to be plotted as a point in a five-dimensional space with axes corresponding to age, number of books read per week, average math test score, number of minutes spent reading the newspaper per day, and number of owls she owns. However, I'm having trouble drawing such a graph. It would require a five-dimensional space, which is very hard to draw. Even computer programs only have 3D graphics.

Nonetheless, in an abstract sense, there exists a five-dimensional space with a collection of five numbers, such as (11, 3, 100, 45, 4), which tells us that Athena is eleven, that she reads three books on the average each week, that she never gets a math question wrong, that she reads the newspaper for forty-five minutes each day, and that she has four owls at the moment. With these five numbers, I've described Athena. If you knew her, you could recognize her from this point in five dimensions.

The number of dimensions for each of the three people above was the number of attributes I used to identify them: one for Sam, three for Ike, and five for Athena. Real people, of course, are generally more difficult to capture with so few items of information.

In the following chapters, we'll use dimensionality to explore not people, but space itself. By "space" I mean the region in which matter exists and physical processes take place. A *space of a particular dimension* is a space requiring a particular number of quantities to specify a point. In one dimension, that would be a point on a plot with a single $x$ axis; in two dimensions, a point on a plot with an $x$ and a $y$ axis; in three dimensions, it would be a point on a plot with an $x$, a $y$, and a $z$ axis.[1,*] Those axes are shown in Figure 5.

In three-dimensional space, three numbers are all you ever need to know your precise location. The numbers you specify might be latitude, longitude, and altitude; or length, width, and height; or you might have a different way to choose your three numbers. The critical thing is that three dimensions means you need precisely three numbers.

*This and other superscript numbers (1, 2, . . . ) refer to the Math Notes at the end of the book.



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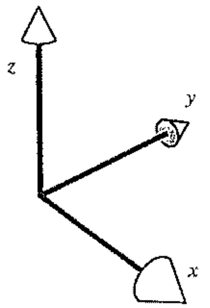


Figure 5. *The three coordinate axes that we use for three-dimensional space.*

In two-dimensional space you need two numbers, and in higher-dimensional space you need more.

More dimensions means freedom to move in a greater number of completely different directions. A point in a four-dimensional space simply requires one additional axis—again, difficult to draw. But it should not be hard to imagine its existence. We'll think about it using words and mathematical terms.

String theory suggests even more dimensions: it postulates six or seven extra spatial dimensions, meaning that six or seven additional coordinates are needed to plot a point. And very recent work in string theory has shown that there could be even more dimensions than that. In this book, I'll keep an open mind and entertain the possibility of any number of extra dimensions. It is too soon to say how many dimensions the universe actually contains. Many of the concepts about extra dimensions that I will describe apply to any number of extra dimensions. In the rare cases when that isn't true, I will make sure that it is clear.

Describing a physical space involves more than just identifying points, however. You need also to specify a *metric*, which establishes the measurement scale, or the physical distance between two points. These are the markings along the axis of a graph. It's not enough to know that the distance between a pair of points is 17 unless you know whether 17 means 17 centimeters, 17 miles, or 17 light-years. A metric is required to tell us how to measure distance: what the distance





between two points on a graph corresponds to in the world that the graph represents. A metric gives a measuring rod that reveals your choice of units in order to set the scale, just like on a map, where a half-inch might represent one mile, or as in the metric system, which gives us a meter stick we all agree on.

But that is not all a metric specifies. It also tells us whether space bends or curls around, like the surface of a balloon when it is blown up into a sphere. The metric contains all the information about the shape of space. A metric for curved space tells us about both distances and angles. Just as an inch can represent different distances, an angle can correspond to different shapes. I'll go into this later on when we explore the connection between curved space and gravity. For now, let's just say that the surface of a sphere is not the same as the surface of a flat piece of paper. Triangles on one don't look like triangles on the other, and the difference between these two-dimensional spaces can be seen in their metrics.[2]

As physics has evolved, so has the amount of information stored in the metric. When Einstein developed relativity, he recognized that a fourth dimension—time—is inseparable from the three dimensions of space. Time, too, needs a scale, so Einstein formulated gravity by using a metric for four-dimensional *spacetime*, adding the dimension of time to the three dimensions of space.

And more recent developments have shown that additional spatial dimensions might also exist. In that case, the true spacetime metric will involve more than three dimensions of space. The number of dimensions and the metric for those dimensions is how one describes such a multidimensional space. But before we explore metrics and metrics for multidimensional spaces any further, let's think more about the meaning of the term "multidimensional space."

## *Playful Passages Through Extra Dimensions*

In Roald Dahl's *Charlie and the Chocolate Factory*, Willy Wonka introduced visitors to his "Wonkavator." In his words, "An elevator can only go up and down, but a Wonkavator goes sideways and slantways and longways and backways and frontways and squareways



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and any other ways that you can think of . . ."* Really, what he had was a device that moved in any direction, so long as it was a direction in the three dimensions we know. It was a nice, imaginative idea.

However, the Wonkavator didn't really go any way "you can think of." Willy Wonka was remiss in that he neglected extra-dimensional passages. Extra dimensions are other directions entirely. They are hard to describe, but they may be easier to understand by analogy.

In 1884, to explain the notion of extra dimensions, the English mathematician Edwin A. Abbott wrote a novel called *Flatland*.† It takes place in a fictitious two-dimensional universe—the Flatland of the title—where two-dimensional beings (of various geometric shapes) reside. Abbott shows us why Flatlanders, who live their whole lives in two dimensions—on a table top, for example—are as mystified by three dimensions as people in our world are by the idea of four.

For us, more than three dimensions requires a stretch of the imagination, but in Flatland three dimensions are beyond its inhabitants' comprehension. Everyone thinks it is obvious that the universe holds no more than their two perceived dimensions. Flatlanders are as insistent about this as most people here are about three.

The book's narrator, A. Square (the namesake of the author, Edwin $A^2$), is introduced to the reality of a third dimension. In the first stage of his education, while he is still confined to Flatland, he watches a three-dimensional sphere travel vertically through his two-dimensional world. Because A. Square is confined to Flatland, he sees a series of disks that increase and then decrease in size, which are slices of the sphere as it passes through A. Square's plane (see Figure 6).

This is initially perplexing to the two-dimensional narrator, who has never imagined more than two dimensions and has never contemplated a three-dimensional object like a sphere. It is not until A. Square has been lifted out of Flatland into the surrounding three-dimensional world that he can truly imagine a sphere. From his new perspective, he recognizes the sphere as the shape made by gluing together the two-dimensional slices he witnessed. Even in his two-dimensional

*Roald Dahl, *Charlie and the Chocolate Factory* (London: Puffin Books, 1998).
†The full title is *Flatland: A Romance of Many Dimensions*.





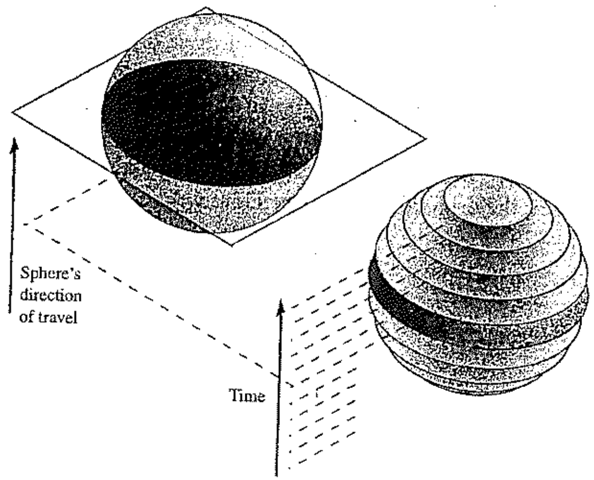


Figure 6. *If a sphere passes through a plane, a two-dimensional observer would see a disk. The sequence of disks that the observer sees over time comprises the sphere.*

world, A. Square could have plotted the disks he sees as a function of time (as in Figure 6) to construct the sphere. But it wasn't until his trip through a third dimension opened his eyes that he fully comprehended the sphere and its third spatial dimension.

By analogy, we know that if a *hypersphere* (a sphere with four spatial dimensions) were to pass through our universe, it would appear to us as a time sequence of three-dimensional spheres that increase, then decrease, in size.[3] Unfortunately, we don't have the opportunity to journey through an extra dimension. We will never see a static hypersphere in its entirety. Nonetheless, we can make deductions about how objects look in spaces of different dimensions—even dimensions that we don't see. We can confidently deduce that our perception of a hypersphere passing through three dimensions would look like a series of three-dimensional spheres.

As another example, let's imagine the construction of a *hypercube*—a generalization of a cube to more than three dimensions. A line segment of one dimension consists of two points connected



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by a straight, one-dimensional line. We can generalize this in two dimensions to a square by putting one of these one-dimensional line segments above another and connecting them with two additional segments. We can generalize further in three dimensions to a cube, which we can construct by placing one two-dimensional square above the other and connecting them with four additional squares, one on each edge of the original squares (see Figure 7).

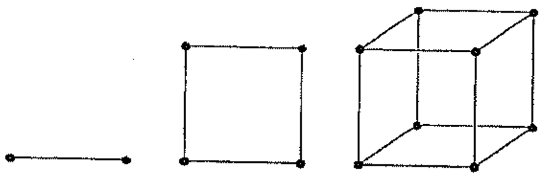

Figure 7. *How we put together lower-dimensional objects to make higher-dimensional ones. We connect two points to make a line segment, two line segments to make a square, two squares to make a cube, and (not pictured since it's too difficult to draw) two cubes to make a hypercube.*

We can generalize in four dimensions to a hypercube, and in five dimensions to something for which we don't yet have a name. Even though we three-dimensional mortals have never seen these two objects, we can generalize the procedure that worked in lower dimensions. To construct a hypercube (also known as a tesseract), put one cube above the other, and connect them by adding six additional cubes, connecting the faces of the two original cubes. This construction is an abstraction and difficult to draw, but that doesn't make the hypercube any less real.

In high school, I spent a summer at math camp (which was far more entertaining than you might think), where we were shown a film version of *Flatland.** At the end, the narrator, in a delightful British accent, tried futilely to point to the third dimension that was inaccessible to Flatlanders, saying, "Upward, not Northward." Unfortunately, we have the same frustration if we try to point to a fourth spatial

*This animated film, directed by Eric Martin, featured the voices of Dudley Moore and other members of the British theatrical comedy group *Beyond the Fringe*. It was very entertaining.





dimension, a passage. But just as Flatlanders didn't see or travel through the third dimension, even though it existed in Abbott's story, our not having yet seen another dimension doesn't mean there is none. So although we haven't yet observed or traveled through such a dimension, the subtext throughout *Warped Passages* will be, "Not Northward, but Forward along a passage." Who knows what exists that we haven't yet seen?

## *Three from Two*

For the rest of this chapter, rather than thinking about spaces that have more than three dimensions, I will talk about how, with our limited visual capacity, we go about thinking and drawing three dimensions using two-dimensional images. Understanding how we perform this translation from two-dimensional images to three-dimensional reality will be useful later on when interpreting lower-dimensional "pictures" of higher-dimensional worlds. Think of this section as a warm-up exercise for wrapping your mind around extra dimensions. It might be good to remember that you cope with dimensionality all the time in ordinary life. It really isn't that unfamiliar.

Often all we can see are parts of the surface of things, the surface being only the exterior. This exterior has two dimensions, even though it curves through three-dimensional space, because you only need two numbers to identify any point. We deduce that the surface isn't three-dimensional because it has no thickness.

When we look at pictures, movies, computer screens, or the figures in this book, we are generally looking at two-dimensional, not three-dimensional representations. But we can nonetheless deduce the three-dimensional reality that is being portrayed.

We can use two-dimensional information to construct three dimensions. This involves suppressing information in making two-dimensional representations while trying to keep enough information to reproduce essential elements of the original object. Let's now reflect on the methods we often use to reduce higher-dimensional objects to lower dimensions—slicing, projection, holography, and sometimes



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just ignoring the dimension—and how we work backwards to deduce the three-dimensional objects they represent.

The least complicated way of seeing beyond the surface is to make slices. Each slice is two-dimensional, but the combination of the slices forms a real three-dimensional object. For example, when you order ham at the deli, the three-dimensional lump of ham is readily exchanged for many two-dimensional slices.* By stacking all the slices you could reconstruct the full three-dimensional shape.

This book is three-dimensional. However, its pages have only two dimensions. The union of the two-dimensional pages comprises the book.† We could illustrate this union of pages in many ways. One is shown in Figure 8, in which we view the book edge on. In this picture we've again played with dimensionality, since each line represents a page. So long as we all know that the lines represent two-dimensional pages, this illustration should be clear. Later on, we'll use a similar shorthand when we depict objects in multi-dimensional worlds.

Slicing is only one way to replace higher dimensions with lower

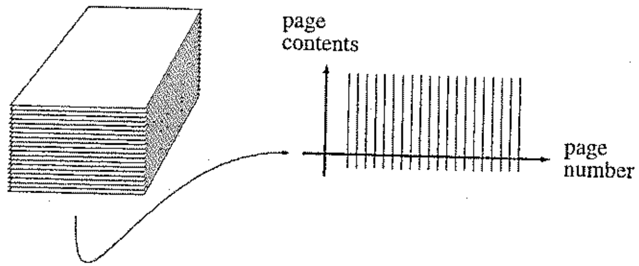


Figure 8. *A three-dimensional book is made up of two-dimensional pages.*

*Slices of ham do have some thickness, so they are in reality thin, but three-dimensional. Their size in this extra dimension is so small that it is a good approximation to think of them as two-dimensional. However, even with arbitrarily thin two-dimensional slices, we can imagine putting them together to make a three-dimensional object in this way.

†Again, for the pages to be truly two-dimensional they would have to be infinitely thin slices with no thickness at all in the third dimension. For now, though, two dimensions is a fine approximation for pages as thin as these.





ones. *Projection*, a technical term borrowed from geometry, is another. A projection gives a definite prescription for creating a lower-dimensional representation of an object. A shadow on a wall is an example of a two-dimensional projection of a three-dimensional object. Figure 9 illustrates how information is lost when we (or rabbits) make a projection. Points on the shadow are identified by only two coordinates, left-right or up-down along the wall. But the object that is projected also has a third spatial dimension that the projection doesn't retain.

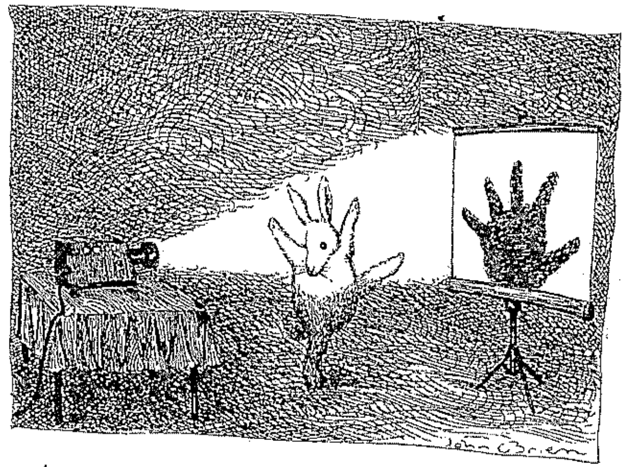

Figure 9. *A projection carries less information than the higher-dimensional object.*

The simplest way to make a projection is to just ignore one dimension. For example, Figure 10 shows a cube in three dimensions being projected onto two dimensions. The projections can take many forms, the simplest of which is a square.

To return to our earlier examples of the graphs of Ike and Athena, we might make a two-dimensional plot of Ike by neglecting his driving fast cars. And we might not really want to know the number of owls Athena raises, and might therefore make a four-dimensional rather than a five-dimensional plot. Disregarding Athena's owls is a projection.



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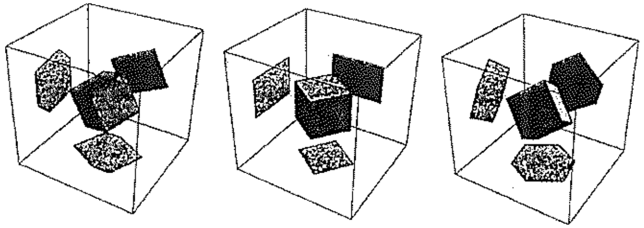

Figure 10. *Projections of a cube. Notice that the projection can be a square, as we see in the middle diagram, but that projections can also take other shapes.*

A projection discards information from the original, higher-dimensional object (see Figure 9). However, when we make a lower-dimensional picture using a projection, we sometime include information to help retain some of what was lost. The additional information might be shading or color, as in a painting or photograph. It might be a number, as in a topographic map to illustrate height. Or there might be no label at all, in which case the two-dimensional characterization simply offers less information.

Without both our eyes, which work together to let us reconstruct three dimensions, everything we see would be projections. Depth perception is tougher when you close one of your eyes. A single eye constructs a two-dimensional projection of three-dimensional reality. You need two eyes to reproduce three dimensions.

I am nearsighted in one eye and farsighted in the other, so I don't properly combine the images from both eyes unless I'm wearing glasses—which is rarely the case. Although I was told I should have trouble reconstructing three dimensions, I don't usually notice any problem: things still look three-dimensional to me. That is because I rely on shading and perspective (and my familiarity with the world) to reconstruct three-dimensional images.

But one day in the desert, a friend and I were trying to reach a distant cliff. My friend kept telling me that we could walk directly there, and I couldn't understand why he was insisting that we should walk straight through a piece of rock. It turned out the rock that I thought projected directly from the cliff, so that it would completely block our way, was in fact located much closer to us, in front of the





cliff. The rock I had thought would bar our path wasn't actually attached to the cliff at all. This misunderstanding occurred because we were near the cliff around noon, when there were no shadows, and I had no way to construct the third dimension that would have told me how the distant cliffs and rocks were lined up. I wasn't really conscious of my compensating strategy of using shading and perspective until then, when it failed.

Painting and drawing have always required artists to reduce what they see to projected images. Medieval art did this in the simplest manner. Figure 11 shows a mosaic image of a city as a two-dimensional projection. This mosaic doesn't tell us anything about a third dimension; there are no labels or indications of its existence.

Since medieval times, painters have developed ways to make projections that partially redress painting's loss of a dimension. One approach that opposes the medieval flattening of space is the method used by the cubists in the twentieth century. A cubist painting (for example Picasso's *Portrait of Dora Maar*, Figure 12) presents several projections simultaneously, each from a different angle, and thereby conveys the subject's three-dimensionality.


Figure 11. *A two-dimensional medieval mosaic.*



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Figure 12. Portrait of Dora Maar, *a cubist painting by Picasso.*

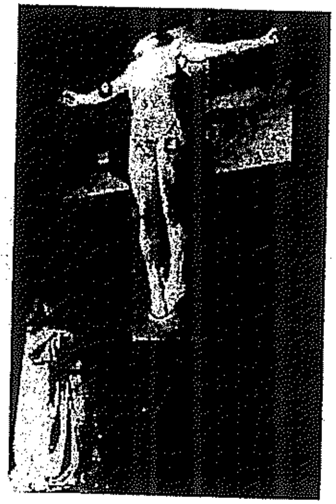

Figure 13. *Dali's* Crucifixion (Corpus Hypercubus).





Most Western painters since the Renaissance, however, have used perspective and shading to create the illusion of a third dimension. One of the essential skills in painting is the ability to reduce a three-dimensional world to a two-dimensional representation that allows the observer to reverse the process and reconstitute the initial three-dimensional scene or object. We are acculturated to know how to decode the images, even though not all of the three-dimensional information is there.

Artists have even tried representing higher-dimensional objects on two-dimensional surfaces. For example, Salvador Dali's *Crucifixion (Corpus Hypercubus)* (see Figure 13) shows the cross as an opened-up hypercube. A hypercube consists of eight cubes attached in four-dimensional space. These are the cubes he has drawn. I've shown a few projections of a hypercube in Figure 14.

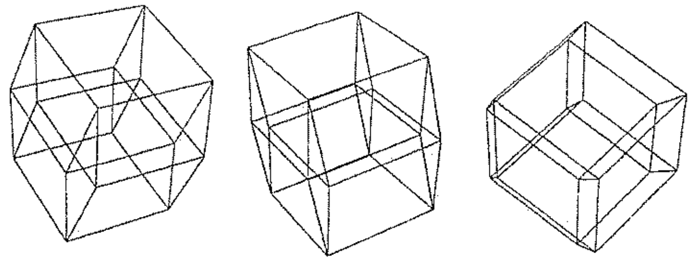

Figure 14. *Projections of a hypercube.*

I have already mentioned a physics example: quasicrystals, which look like the projection of a higher-dimensional crystal into our three-dimensional world. Projections can also be used for practical, not just artistic purposes. Medicine contains many examples where three-dimensional objects are projected onto two dimensions. An X-ray always records a two-dimensional projection. CAT (computer-assisted tomography) scans combine multiple X-ray images to reconstruct a more informative three-dimensional representation. With X-rays taken from sufficiently many angles, one can use interpolation to piece together full three-dimensional images. An MRI (magnetic



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resonance imaging) scan, on the other hand, reconstructs a three-dimensional object from slices.

A holographic image is another way to record three dimensions on a two-dimensional surface. Although a holographic image is recorded on a lower-dimensional surface, it actually carries all the information of the original higher-dimensional space. You probably have an example of this technique in your wallet: the three-dimensional-looking image on your credit card is a hologram.

A holographic image records relationships between light in different places, so that the full higher-dimensional image can be recovered. This principle is much the same as that used in a good stereo, which lets you hear where instruments were being played in relation to each other when they were recorded. With the information stored in a hologram, the eye can truly reconstruct the three-dimensional object it represents.

These methods tell us how we might get more information from a lower-dimensional image. But maybe all you really need is less information. Sometimes you just don't care about all three dimensions. For example, something might be so thin in the third dimension that nothing interesting happens in this direction: even though the ink on this paper is really three-dimensional, we lose nothing by thinking of it as two-dimensional. Unless we look at the page under a microscope, we simply don't have the necessary resolution to see the ink's thickness. A wire looks one-dimensional even though, on closer inspection, you can see it has a two-dimensional cross-section and therefore three dimensions in all.

## *Effective Theories*

There is nothing wrong with ignoring an extra dimension that's too small to be seen. Not only the visual effects, but also the physical effects of tiny, undetectable processes can usually be ignored. Scientists often average over or ignore (often unwittingly) physical processes that occur on immeasurably small scales when formulating their theories or setting up their calculations. Newton's laws of motion work at the distances and speeds he could observe. He didn't need the





details of general relativity to make successful predictions. When biologists study a cell, they don't need to know about quarks inside the proton.

Selecting relevant information and suppressing details is the sort of pragmatic fudging everyone does every day. It's a way of coping with too much information. For almost anything you see, hear, taste, smell, or touch, you have the choice between examining details by scrutinizing very closely, and looking at the "big picture" with its other priorities. Whether you are staring at a painting, tasting wine, reading philosophy, or planning your next trip, you automatically parcel your thoughts into the categories of interest—be they sizes or flavors or ideas—and the categories that you don't find relevant at the time. When appropriate, you ignore some details so that you can focus on the issue of interest, and not obscure it with inessential details.

This procedure of disregarding small-scale information should be familiar because it's actually a conceptual leap people make all the time. Take New Yorkers, for example. New Yorkers living in the thick of the city see the details and variation within Manhattan. To them, downtown is funkier, older, with narrower, more crooked streets. Uptown has more real estate that was designed for human beings to actually live in, as well as Central Park and most of the museums. Although such distinctions are blurred from far away, within the city they are very real.

But now think about how people far away see New York. To them, it's a dot on a map. An important dot, perhaps, a dot with a distinctive character; but from outside New York, a dot nonetheless. Even with all their variety, New Yorkers are in a single category when viewed from the Midwest or Kazakhstan, for example. When I mentioned this analogy to my cousin who lives downtown (in the West Village, to be precise), he confirmed my point by balking at the suggestion of grouping together New Yorkers living uptown and downtown. Nonetheless, as any non-New Yorker could tell him, the distinctions are too small to matter to people not living in their midst.

It is common practice in physics to formalize this intuition, and organize categories in terms of the distance or energy that is relevant. Physicists accept this practice and have given it a name—*effective theory*. The effective theory concentrates on the particles and forces that have "effects" at the distance in question. Rather than describing particles and interactions in terms of unmeasurable parameters that describe ultra-high-energy behavior, we formulate observations in terms of the things that are actually relevant to the scales we might detect. The effective theory at any single distance scale doesn't go into the details of an underlying short-distance physical theory; it only asks about things you could hope to measure or see. If something is beyond the resolution of the scales at which you are working, you don't need its detailed structure. This practice is not scientific fraud, but a way of disregarding the clutter of superfluous information. It is an "effective" way to obtain accurate answers efficiently.

Everyone, including physicists, is happy to return to a three-dimensional universe when higher-dimensional details are beyond our resolution. Just as physicists will often treat a wire as if it is one-dimensional, we will also describe a higher-dimensional universe in lower-dimensional terms when the extra dimensions are minuscule and higher-dimensional details are too tiny to matter. Such a lower-dimensional description would summarize the observable effects of all possible higher-dimensional theories in which the extra dimensions are too tiny to see. For many purposes, such a lower-dimensional description is adequate, independent of the number, size, and shape of the additional dimensions.

The lower-dimensional quantities are not providing the fundamental description, but they are a convenient way of organizing observations and predictions. If you do know the short-distance details, or the microstructure, of a theory, you can use them to derive the quantities that appear in the low-energy description. Otherwise, those quantities are just unknowns to be experimentally determined.

The following chapter elaborates these ideas and considers the consequences of tiny rolled-up extra dimensions. The dimensions we'll consider first are minuscule, too tiny to make any difference at all. Later on, when we return to extra dimensions, we'll explore both the large and the infinite dimensions that recently radically revised this picture.



# 2

# Restricted Passages: Rolled-up Extra Dimensions

No way out
None whatsoever.

Jefferson Starship

*Athena awoke with a start. The previous day she had read* Alice in Wonderland *and* Flatland *in order to seek some inspiration about dimensions. But that night she had the strangest dream, which, when fully conscious, she recognized as the result of having read the two books on the same day.**

*Athena dreamed she had turned into Alice, slipped into a rabbit hole, and met the resident Rabbit, who had pushed her out into an unfamiliar world. Athena had thought it a rather rude way to convey a guest. Even so, she had eagerly looked forward to her upcoming adventure in Wonderland.*

*Athena was in for a disappointment, however. The resident Rabbit, who was fond of puns, had sent her instead to OneDLand, a strange, not so wonderful, one-dimensional world. Athena looked around—or, I should say, to her left and right—and discovered that all she could see were two points—one to her left and another to her right (but in a prettier color, she thought).*

*In OneDLand, all the one-dimensional people with their one-dimensional possessions were lined up along this single dimension like long, thin beads strung out along a thread. But even with her limited*

*Or perhaps this story is a result of my having begun my education at the perhaps questionably named Lewis Carroll School, P.S. 179, in Queens.



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